Exercise 11

A machinist is required to manufacture a circular metal disk with area 1000 cm^2.

(a) What radius produces such a disk?

(b) If the machinist is allowed an error tolerance of ± 5 cm^2 in the area of the disk, how close to the ideal radius in part (a) must the machinist control the radius?

(c) In terms of the ε, δ definition of $\lim_{x\to a} f(x) = L$, what is x? What is $f(x)$? What is a? What is L? What value of ε is given? What is the corresponding value of δ?

Solution

Part (a)

The formula relating the area of a circle with its radius is

$$A = \pi r^2.$$

Set $A = 1000$ and solve for r.

$$1000 = \pi r^2$$

$$\frac{1000}{\pi} = r^2$$

$$r = \sqrt{\frac{1000}{\pi}} \approx 17.84 \text{ cm}$$

Part (b)

With an error tolerance of ± 5 cm^2, there are upper and lower bounds for the area.

$$\text{Lower Bound:} \quad 1000 - 5 = 995 \text{ cm}^2$$

$$\text{Upper Bound:} \quad 1000 + 5 = 1005 \text{ cm}^2$$

Determine the radii corresponding to these areas.

$$995 = \pi r_l^2 \qquad \Rightarrow \qquad r_l = \sqrt{\frac{995}{\pi}} \approx 17.80 \text{ cm}$$

$$1005 = \pi r_u^2 \qquad \Rightarrow \qquad r_u = \sqrt{\frac{1005}{\pi}} \approx 17.89 \text{ cm}$$

Therefore, the error tolerance for the radius of the disk is

$$\pm\frac{r_u - r_l}{2} = \pm\frac{1}{2}\left(\sqrt{\frac{1005}{\pi}} - \sqrt{\frac{995}{\pi}}\right) \approx \pm 0.0446 \text{ cm}.$$

Part (c)

In terms of the ε, δ definition of

$$\lim_{x \to a} f(x) = L,$$

x is the radius r, $f(x)$ is the area $A(r)$, a is the ideal radius $\sqrt{1000/\pi}$ corresponding to the desired 1000 cm^2 area, L is the desired area 1000 cm^2, ε is the provided error tolerance of 5 cm^2 for the area, and δ is the error tolerance of about 0.0446 cm for the radius.